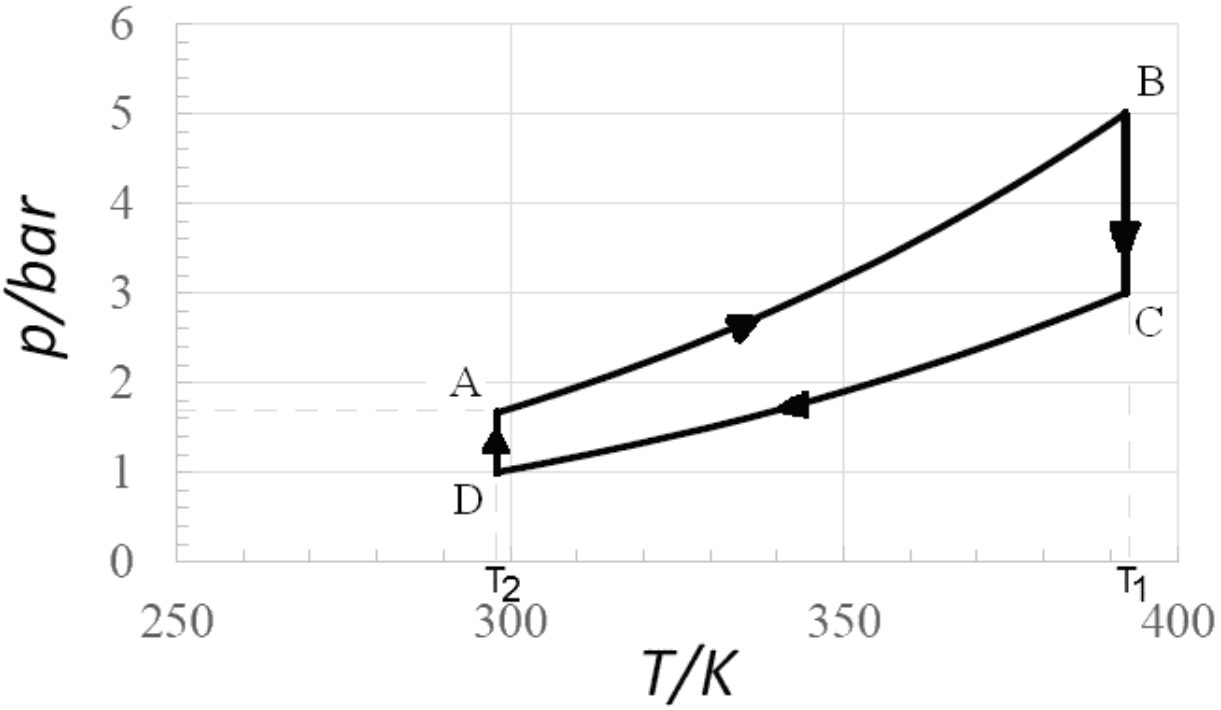
Problema 3 Carnot cycle



These data are read from the graphic:
Point A) P_A = 1.667 bar; T_2 = 298 K Point B) P_B = 5.0 bar; T_1 = 392 K
Point D) P_D = 1.0 bar; T_2 = 298 K Point C) P_C = 3.0 bar; T_1 = 392 K

3.1 In a thermodynamic cycle $\Delta U = 0 \Rightarrow Q = W$
in the adiabatic compression AB: $Q_{AB} = 0$
in the isothermal expansion BC: $Q_{BC} = W_{BC} = n\ R\ T_1 \ln(V_C/V_B) = n\ R\ T_1 \ln(P_B/P_C) =$
$= n\ mol\ 8{,}314\ J/(mol\ K)\ 392\ K \ln(5/3) = 1665\ n\ J$
in the adiabatic expansion CD: $Q_{CD} = 0$
in the isothermal compression DA: $Q_{DA} = W_{DA} = n\ R\ T_2 \ln(V_A/V_D) = n\ R\ T_2 \ln(P_D/P_A) =$
$= n\ mol\ 8{,}314\ J/(mol\ K)\ 298\ K\ \ln(1/1{,}667) = -1266\ n\ J$
$W = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = 399\ n\ J$
The magnitude of useful work is not definable because the quantity of substance (n) is missing.

3.2 Generally the efficiency η of a thermodynamic cycle is defined as:
η = (useful work)/(given heat) = W/Q_{BC} = (399 n *J*)/(1665 n *J*) = 0,24
even, for Carnot cycle only, the efficiency is:
$\eta = 1 - (T_2/T_1) = 1 - (298\ K/392\ K) = 0{,}24$

3.3 In the adiabatic expansion CD: $P_C\ V_C^{\gamma} = P_D\ V_D^{\gamma} \Rightarrow P_C\ (T_1/P_C)^{\gamma} = P_D\ (T_2/P_D)^{\gamma} \Rightarrow$
$P_C^{1-\gamma}\ T_1^{\gamma} = P_D^{1-\gamma}\ T_2^{\gamma} \Rightarrow (P_C/P_D)^{1-\gamma} = (T_2/T_1)^{\gamma} \Rightarrow (1-\gamma) \ln(P_C/P_D) = \gamma \ln(T_2/T_1) \Rightarrow$
$1/\gamma - 1 = \ln(T_2/T_1)/\ln(P_C/P_D) \Rightarrow 1/\gamma = \ln(T_2/T_1)/\ln(P_C/P_D) + 1 \Rightarrow$
$1/\gamma = \ln(298/392)/\ln(3/1) + 1 = 0{,}75 \Rightarrow \gamma = 1/0{,}75 = 1/(3/4) = 1{,}33$
$\gamma = 1 + R/C_V \Rightarrow \gamma - 1 = R/C_V \Rightarrow C_V = R/(\gamma - 1) = R/(1{,}33 - 1) \Rightarrow C_V = 3\ R$

3.4 Remembering that, for an ideal gas, $C_{V\ monoatomic} = 3/2\ R$, $C_{V\ diatomic} = 3/2\ R + 2/2\ R = 5/2\ R$, $C_{V\ triatomic} = 3/2\ R + 3/2\ R = 3\ R$, we can affirm that this molecule with $C_V = 3\ R$ is a triatomic non-linear gas molecule.

Solution proposed by Roberto Tinelli – Student at ITIS "L. dell'Erba" – Castellana Grotte – Italy