



## 54<sup>th</sup> INTERNATIONAL CHEMISTRY OLYMPIAD 2022 UK Round One MARK SCHEME

Although we would encourage students to always quote answers to an appropriate number of significant figures, do not penalise students for significant figure errors. Allow where a student's answers differ slightly from the mark scheme due to the use of rounded/non-rounded data from an earlier part of the question.

In general, 'error carried forward' (referred to as ECF) can be applied. We have tried to indicate where this may happen in the mark scheme and where ECF is not allowed.

For answers with missing or incorrect units, penalise one mark for the first occurrence in **each** question and write **UNIT** next to it. Do not penalise for subsequent occurrences in the same question.

Organic structures are shown in their skeletal form, but also accept displayed formulae as long as the representation is unambiguous.

State symbols are not required for balanced equations and students should not be penalised if they are absent.

No half marks are to be awarded. One blank tick box has been included per mark available for each part. Please mark by placing a tick in each box if mark is scored.

Question	1	2	3	4	5	6	Total
Marks Available	9	9	24	13	19	12	86

1.	This question is about E10 petrol	Mark
(a)	Only accept skeletal formula.	
(b)	$6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$ Must be fully correct for mark.	V
(c)	$C_6H_{12}O_6 \rightarrow 2C_2H_5OH + 2CO_2$ Must be fully correct for mark. Accept if ethanol written as $C_2H_6O$ .	N
(d)	$C_8H_{18} + 12.5O_2 \rightarrow 8CO_2 + 9H_2O$ Total bond enthalpy of reactants = [(7 × 347) + (18 × 413) + (12.5 × 498)] kJ mol <sup>-1</sup> = 16,088 kJ mol <sup>-1</sup> Total bond enthalpy of products = [(16 × 805) + (18 × 464)] kJ mol <sup>-1</sup> = 21,232 kJ mol <sup>-1</sup> Enthalpy of combustion = [16088 - 21232] kJ mol <sup>-1</sup> = -5144 kJ mol <sup>-1</sup> <i>Must be negative for mark. Must be scaled for one mole of C</i> <sub>8</sub> H <sub>18</sub> ( <i>i.e., not based on a balanced equation with 2C</i> <sub>8</sub> H <sub>18</sub> and 25O <sub>2</sub> ).	Ŋ
(e)	Ethanol: $-1276 \text{ kJ mol}^{-1} / 46.07 \text{ g mol}^{-1} = -27.70 \text{ kJ g}^{-1}$ $-27.70 \text{ kJ g}^{-1} \times 0.789 \text{ g cm}^{-3} = -21.86 \text{ kJ cm}^{-3}$ Octane: $-5144 \text{ kJ mol}^{-1} / 114.22 \text{ g mol}^{-1} = -45.04 \text{ kJ g}^{-1}$ $-45.04 \text{ kJ g}^{-1} \times 0.703 \text{ g cm}^{-3} = -31.66 \text{ kJ cm}^{-3}$ Energy released from 1 litre (1000 cm <sup>3</sup> ) of E5 fuel = (950 cm <sup>3</sup> × 31.66 kJ cm}^{-3}) + (50 cm <sup>3</sup> × 21.86 kJ cm}^{-3}) = 31170 kJ Energy released from 1 litre (1000 cm}^{3}) of E10 fuel = (900 cm}^{3} × 31.66 kJ cm}^{-3}) + (100 cm}^{3} × 21.86 kJ cm}^{-3}) = 30680 kJ Both correct energy released values scores all three marks. If calculation done via route above, then the first mark is awarded for the value of $-21.86 \text{ kJ cm}^{-3}$ for octane (or 21.86 kJ cm}^{-3}). The second mark is of both correct energy released values. A total of one out of three can be given if the ethanol and octane values have been calculated incorrectly but combined in the correct proportions for E5 and E10. Alternatively, students may use a different route. In 1 litre of E5 fuel Amount of ethanol = 50 cm}^{3} × 0.789 g cm}^{-3} / 46.07 g mol}^{-1} = 0.8563 mol Amount of octane = 950 cm}^{3} × 0.703 g cm}^{-3} / 114.22 g mol}^{-1} = 5.847 mol Energy released = (0.8563 mol × 1276 kJ mol}^{-1}) + (5.847 mol × 5144 kJ mol}^{-1}) = 31170 kJ In 1 litre of E10 fuel	Image: Constraint of the second secon

	Total out of 9	9
(g)	90/95 × 100 = 94.7%	$\mathbf{V}$
(f)	30680 kJ / 31170 kJ × 100 = 98.4% Allow ECF for incorrect energy values calculated in part (e).	$\mathbf{V}$
	Allow ECF for incorrect value of enthalpy of combustion of octane from part (d), or where the value of $-6666$ kJ mol <sup>-1</sup> has been used.	
	Both correct energy released values scores all three marks. If calculation done via this route, then the first mark is awarded if the moles of ethanol and octane in 1 litre of E5 fuel are both correct. The second mark is awarded if the moles of ethanol and octane in 1 litre of E10 fuel are both correct. The third mark is for both correct energy released values.	
	= 30680 kJ	
	Energy released = (1.713 mol × 1276 kJ mol <sup>-1</sup> ) + (5.539 mol × 5144 kJ mol <sup>-1</sup> )	
	Amount of octane = 900 cm <sup>3</sup> × 0.703 g cm <sup>-3</sup> / 114.22 g mol <sup>-1</sup> = 5.539 mol	
	Amount of ethanol = 100 cm <sup>3</sup> × 0.789 g cm <sup>-3</sup> / 46.07 g mol <sup>-1</sup> = 1.713 mol	









4.	This question is about coronavirus testing			Mark	
(a)	Make a very acidic solution	Make a neutral solution	Make a very alkaline solution	Make a buffered ✓ solution	
(b)	Very Acidic 🗸	Neutral	pH 7.4	Very Alkaline	
	Note in this case the t the acid overwhelms t	case the buffer concentration is much lower than the acid concentration, so rwhelms the buffer.			
(c)	7.1 × 10 <sup>6</sup> virus particle [virus particles] = num 7.1 × 10 <sup>9</sup> virus particle = 1.18 × 10 <sup>-14</sup> mol dm [SP] = 20 × [virus parti = 2.36 × 10 <sup>-13</sup> mol dm	$\begin{array}{l} 1 \times 10^{6} \mbox{ virus particles / cm}^{3} \times (1000 \mbox{ cm}^{3} / \mbox{ dm}^{3}) = 7.1 \times 10^{9} \mbox{ virus particles / dm}^{3} \label{eq:virus particles} = number of virus particles / N_{A} = \\ 1 \times 10^{9} \mbox{ virus particles / dm}^{3} / \mbox{ 6.02 } \times 10^{23} \mbox{ mol}^{-1} \label{eq:virus particles} = 1.18 \times 10^{-14} \mbox{ mol dm}^{-3} \label{eq:virus particles} = 20 \times 1.18 \times 10^{-14} \mbox{ mol dm}^{-3} \label{eq:virus particles} = 2.36 \times 10^{-13} \mbox{ mol dm}^{-3} \end{array}$			
(d)	$[NP]_0 = 1.6 \times 10^{12} \text{ cm}^-$	<sup>3</sup> × (1000 cm <sup>3</sup> / dm <sup>3</sup> ) / I	N <sub>A</sub> = 2.66 × 10 <sup>-9</sup> mol dr	n <sup>-3</sup>	
	Species	Original Concentra	tion After Equilibri	um Concentration	
	SP	$2.36  imes 10^{-13}$ mol dn	n <sup>-3</sup> 2.36 × 10 <sup>-</sup>	$^{13}$ mol dm $^{-3} - x$	
	NP	$2.66  imes 10^{-9}$ mol dn	n <sup>-3</sup> 2.66 × 10 <sup>-</sup>	<sup>.9</sup> mol dm <sup>-3</sup> − <i>x</i>	
	SPNP	0		<i>x</i>	
	$x \leq [SP]_0 \text{ so any chan}$ $K = \frac{[SPNP]}{[NP][SP]} = 1.2 \times 10^{10}$ $\approx \frac{x}{[NP]_0([SP]_0 - x)}$ $x = K[NP]_0([SP]_0 - x)$ $x(1 + K[NP]_0) = K[NR]$ $x = \frac{K[NP]_0}{(1 + K[NP]_0)}[SP]_0 = K[NR]$	ge in [NP] is small, so [ $p^{10} \text{ mol}^{-1} \text{ dm}^3$ $p^{0}_0[SP]_0$ $0.97 [SP]_0 = 2.29 \times 10^{-1}$ cores both marks. One vations. Allow ECF from mol dm <sup>-3</sup> if [SP] = 1.0 ×	NP] $\approx$ [NP] <sub>0</sub> $\approx$ 2.66 $\times$ 1 <sup>13</sup> mol dm <sup>-3</sup> <sup>a</sup> mark for correct [NP] <sub>0</sub> <sup>b</sup> part (c). Answer = 0.9 10 <sup>-11</sup> mol dm <sup>-3</sup> .	$10^{-9}$ mol dm <sup>-3</sup> and one <i>mark for</i> $10^{-9}$ × (answer from (c)).	
(e)	(i) Area of strip A = Number of AB =	$3 \text{ mm}^2$ . $3 \text{ mm}^2 \times 1.2 \times 10^9 \text{ mm}^3$	$^{-2} = 3.6 \times 10^{9}.$		V
	(ii) Volume of strip Amount of SPNF Number of SPNF	$V = 0.3 \text{ mm}^3 = 3 \times 10^{-7}$ = 2.29 × 10 <sup>-13</sup> mol dm <sup>-13</sup> = 6.86 × 10 <sup>-20</sup> mol × 6	dm <sup>3</sup> $^{-3} \times 3 \times 10^{-7}  dm^3 = 6.86$ $.02 \times 10^{23}  mol^{-1} = 4.13$	5 × 10 <sup>−20</sup> mol × 10 <sup>4</sup> .	<b>V</b>

	$[SP] = [SP]_0 - [SPNP] = 7.2 \times 10^{-15} \text{ mol dm}^{-3}$	
	[NB 7 $\times$ 10 <sup>-15</sup> mol dm <sup>-3</sup> if full accuracy not maintained]	
	$ \begin{bmatrix} SP] = [SP]_0 - [SPNP] = 7.2 \times 10^{-16} \text{ mol dm}^{-3} \\ [NB 7 \times 10^{-15} \text{ mol dm}^{-3} \text{ if full accuracy not maintained}] \\ Amount of SP = 7.2 \times 10^{-15} \text{ mol dm}^{-3} \times 3 \times 10^{-7} \text{ dm}^3 = 2.1 \times 10^{-21} \text{ mol} \\ Number of SP = 7.2 \times 10^{-15} \text{ mol dm}^{-3} \times 3 \times 10^{-7} \text{ dm}^3 = 2.1 \times 10^{-21} \text{ mol} \\ Number of SP = 2.1 \times 10^{-21} \text{ mol} \times 6.02 \times 10^{23} \text{ mol}^{-1} = 1.3 \times 10^{3}. \\ One mark for number of SPNP and one mark for number of SPNP = 1.8 \times 10^{3}. \\ One mark for number of SP = 1.8 \times 10^{17} \times [SP], and number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 9.7 \times 10^{-12} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 1.0 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 1.0 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 0.7 \times 10^{-12} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 0.7 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 0.7 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 0.7 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ number of SP = 0. \\ \text{If using } [SPNP] = 0.7 \times 10^{-11} \text{ mol dm}^{-3}, number of SPNP = 1.8 \times 10^{5}, \\ \frac{\Delta_{ABSPNP}}{\sigma_{AB}} = [SPNP] K \\ \frac{\Delta_{ABSPNP}}{\sigma_{AB}} = [SPNP] K \\ \frac{\sqrt{V}{A \sigma_{AB}}} = [SPNP] K \\ \frac{\sqrt{V}{V \sigma_{AB}}} = [SPNP] N \\ \frac{\sqrt{V}{V \sigma_{AB}}} = [SPNP]_{0} \times \sigma_{AB} \\ \text{We note that } 1 \ll KA \sigma_{AB}/V \text{, so} \\ y \approx [SPNP]_{0} \text{ V} \\ \text{For the complete sample passing, we can use V = 0.10 \text{ cm}^{3} \\ y = 2.29 \times 10^{-13} \text{ mol dm}^{-3} \times 1 \times 10^{-4} \text{ dm}^{3} \times 6.02 \times 10^{23} \text{ mol}^{-1} \\ = 1.38 \times 10^{7} \text{ ABSPNP} \\ \text{Correct answer scores all three marks. First mark for } 1 \ll KA \sigma_{AB}/V, \text{ second mark for} \\ y \approx [SPNP]_{0}  V, and third mark for final answer. Note the number of AB >> number$	
$[SP] = [SP]_{0} - [SPNP] = 7.2 \times 10^{-15} \text{ mol dm}^{-3}$ $[NB 7 \times 10^{-15} \text{ mol dm}^{-3} \text{ if full accuracy not maintained}]$ Amount of SP = 7.2 × 10 <sup>-15</sup> mol dm <sup>-3</sup> x 3 × 10 <sup>-7</sup> dm <sup>3</sup> = 2.1 × 10 <sup>-21</sup> mol Number of SP = 2.1 × 10 <sup>-21</sup> mol × 6.02 × 10 <sup>23</sup> mol <sup>-1</sup> = 1.3 × 10 <sup>3</sup> . One mark for number of SPNP and one mark for number of SP. Error carried forward can be given. Number of SP = 1.81 × 10 <sup>17</sup> × [SP], and number of SPNP = 1.81 × 10 <sup>17</sup> × [SPNP]. If using [SPNP] = 9.7 × 10 <sup>-12</sup> mol dm <sup>-3</sup> , number of SPNP = 1.8 × 10 <sup>6</sup> , number of SP = 0. If using [SPNP] = 1.0 × 10 <sup>-11</sup> mol dm <sup>-3</sup> , number of SPNP = 1.8 × 10 <sup>6</sup> , number of SP = 0. If using [SPNP] = 1.0 × 10 <sup>-11</sup> mol dm <sup>-3</sup> , number of SPNP = 1.8 × 10 <sup>6</sup> , number of SP = 0. (f) If y SPNP bind to AB to make y ABSPNP then we have: [SPNP]= [SPNP]_0 × y/ V and $\sigma_{ABSPNP} = [SPNP] K$ $\frac{\sigma_{ABSPNP}}{\sigma_{AB}} = [SPNP] K$ $\frac{\gamma}{4 \sigma_{AB}} = ([SPNP]_0 - y/V)K$ $y (1 + KA \sigma_{AB}/V) = [SPNP]_0 KA \sigma_{AB}$ We note that $1 \ll KA \sigma_{AB}/V$ , so $y = [SPNP]_0 V$ For the complete sample passing, we can use $V = 0.10 \text{ cm}^3$ $y = 2.29 \times 10^{-13} \text{ mol dm}^{-3} \times 1 \times 10^{-4} \text{ dm}^{-3} \times 6.02 \times 10^{23} \text{ mol}^{-1}$ $= 1.38 \times 10^7 \text{ ABSPNP}$ Correct answer scores all three marks. First mark for $1 \ll KA \sigma_{AB}/V$ , second mark for $y = [SPNP]_0 V$ . And third mark for final answer. Note the number of $AB >>$ number of $SPNP$ in (e)). (g) This test gave 1.38 \times 10^7 \text{ ABSPNP in 3 mm}^2 so density 4.59 \times 10^6 \text{ mm}^{-2}. This test had $7.1 \times 10^5$ visus particles in test sample. As number of ABSPNP proportional to number of visus particles: Minimum observable number = (3 × 10 <sup>6</sup> / 4.59 \times 10 <sup>6</sup> mm^{-2} and second mark for final answer. Allow error carried forward. Answer = 6.39 \times 10^{12} / (answer to (f))		
	One mark for number of <i>SPNP</i> and one mark for number of <i>SP</i> . Error carried forward can be given. Number of $SP = 1.81 \times 10^{17} \times [SP]$ , and number of $SPNP = 1.81 \times 10^{17} \times [SPNP]$ .	
	If using [SPNP] = $9.7 \times 10^{-12}$ mol dm <sup>-3</sup> , number of SPNP = $1.8 \times 10^6$ , number of SP = 0.	
	If using [SPNP]= $1.0 \times 10^{-11}$ mol dm <sup>-3</sup> , number of SPNP = $1.8 \times 10^6$ , number of SP = 0.	
(f)	If $y$ SPNP bind to AB to make $y$ ABSPNP then we have:	
	[SPNP]= [SPNP] <sub>0</sub> – $y/V$ and $\sigma_{ABSPNP} = y/A$	
	Rearranging the equilibrium constant	
	$\frac{\sigma_{ABSPNP}}{\sigma_{AB}} = [SPNP] K$	
	$\frac{A \sigma_{ABSPNP}}{A \sigma_{AB}} = [SPNP] K$	
	$\frac{y}{A \sigma_{AB}} = ([SPNP]_0 - y/V)K$	$\mathbf{\nabla}$
	$y (1 + KA \sigma_{AB}/V) = [SPNP]_0 KA \sigma_{AB}$	
	We note that $1 \ll KA \sigma_{AB}/V$ , so	
	$y \approx [\text{SPNP}]_0 \text{ V}$	
	For the complete sample passing, we can use $V = 0.10 \text{ cm}^3$	
	$y = 2.29 \times 10^{-13} \text{mol dm}^{-3} \times 1 \times 10^{-4} \text{dm}^3 \times 6.02 \times 10^{23} \text{mol}^{-1}$	
	$= 1.38 \times 10^7 \text{ABSPNP}$	
	Correct answer scores all three marks. First mark for $1 \ll KA \sigma_{AB}/V$ , second mark for $y \approx [SPNP]_0 V$ , and third mark for final answer. Note the number of $AB >>$ number of SPNP, which means that $\sigma_{AB}$ stays large and approximately constant throughout flow and so all SPNP bind. Error carried forward can be given. Answer = 333 × (number of SPNP in (e)). Error carried forward not given if number of SPNP in (e) = 0.	
(g)	This test gave $1.38 \times 10^7$ ABSPNP in 3 mm <sup>2</sup> so density $4.59 \times 10^6$ mm <sup>-2</sup> . This test had $7.1 \times 10^5$ virus particles in test sample.	
	As number of ABSPNP proportional to number of virus particles:	
	Minimum observable number = $(3 \times 10^6 / 4.59 \times 10^6) \times (7.1 \times 10^5)$	
	= $4.64 \times 10^5$ virus particles	
	Correct answer scores both marks. First mark for the density of $4.59 \times 10^6$ mm <sup>-2</sup> and second mark for final answer. Allow error carried forward. Answer = $6.39 \times 10^{12}$ / (answer to (f))	
	Total out of 13	13



	Also accept if Br exchanged for	G Br N Br		R
(e)		NH <sub>2</sub>		
(f)	Contraction of the second seco	atoms coordinated correct!	y two marks.	2
(g)	tetrahedral	trigonal planar	square planar	
	octahedral 🗸	square pyramidal	hexagonal planar	
(h)	$M^{2+} = Fe^{2+}$ and $X^- = BF_4^-$ The iron(II) can be identified from From the loss of mass when hea $6 \times 18.106 = 32.0\%$ , therefore the Mass of two anions = 337.8 – (6) Mass of one anion = 86.657 g m One mark for Fe <sup>2+</sup> . Two marks for single anion as 86.657 g mol <sup>-1</sup> .	In the characteristic ion test ating, the molecular mass can be total mass of complex = $3^{\circ}$ $\times 18.106 + 55.85)$ g mol <sup>-1</sup> = $10^{-1}$ (Mass of BF <sub>4</sub> <sup>-</sup> = 86 for BF <sub>4</sub> <sup>-</sup> . One of these two can	result. an be obtained. 337.8 g mol <sup>-1</sup> = 173.314 g mol <sup>-1</sup> 5.81 g mol <sup>-1</sup> ) an be given for a mass of a	A A A
(i)	<ul> <li>(i) Chinese knots ✓</li> <li>Must be this answer only f</li> </ul>	or mark.		
	<ul> <li>(ii) Individual rings ✓ and Lin</li> <li>Must be these two answer</li> </ul>	near organic molecules ✓ s and no others for mark.		$\mathbf{\nabla}$
	<ul> <li>(iii) Two interlinked rings ✓ a</li> <li>Must be these two answer</li> </ul>	nd Linear organic molecule s and no others for mark.	s 🗸	V
	<ul> <li>(iv) Chinese knots ✓</li> <li>Must be this answer only f</li> </ul>	or mark.		$\mathbf{V}$
			Total out of 19	19

6.	This question is about storing vaccines	Mark
(a)	Process 1: Si(OCH <sub>2</sub> CH <sub>3</sub> ) <sub>4</sub> + 4 H <sub>2</sub> O $\rightarrow$ Si(OH) <sub>4</sub> + 4 CH <sub>3</sub> CH <sub>2</sub> OH Process 2: Si(OH) <sub>4</sub> $\rightarrow$ SiO <sub>2</sub> + 2 H <sub>2</sub> O One mark for each correct equation.	<b>N</b>
(b)	Process 3 = Process 5 – (6 × Process 6) + (6 × Process 4) = $[-100.3 + (6 \times -56.7) - (6 \times -61.5)]$ kJ mol <sup>-1</sup> = $-71.5$ kJ mol <sup>-1</sup> Answer must be negative for mark.	V
(c)	Using the relation between Gibbs free energy and the equilibrium constant, $\Delta G^{\ominus} = \Delta H^{\ominus} - T \Delta S^{\ominus} = -RT \ln K_{eq}$ and $\ln K_{eq} = \ln \frac{1}{[Si(OH)_4]} = -\ln[Si(OH)_4]$ we can rearrange for $\ln[Si(OH)_4] = \frac{\Delta H^{\ominus} - T \Delta S^{\ominus}}{RT} = \frac{\Delta H^{\ominus}}{RT} - \frac{\Delta S^{\ominus}}{R}$ Comparing to the relation $\ln[Si(OH)_4] = -\frac{1680}{T} - 0.605$ we identify $\Delta H^{\ominus} = -1680 \text{ K} \times 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ $= 14.0 \text{ kJ mol}^{-1}$ $\Delta S^{\ominus} = 0.605 \times 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ $= 5.03 \text{ J K}^{-1} \text{ mol}^{-1}$ Both correct answers score all three marks. First mark for the dependence of $\ln[Si(OH)_4] = -\frac{1}{R} + \frac{1}{R} + $	Image: Constraint of the second se
(d)	A = c - 1 = 0.56744	$\mathbf{\nabla}$
(e)	For $t = 1 \ \mu s$ , $f_1 = -0.143809 \ nm^{-1}$ and for $t = 2 \ \mu s$ , $f_2 = -0.140792 \ nm^{-1}$ . $a = \frac{f_2 - f_1}{2 - 1} = 3.017 \times 10^{-3} \ nm^{-1} \ \mu s^{-1}$ $b = a \times 1 - f_1 = 0.146826 \ nm^{-1}$ One mark for correctly calculating each of a and b.	N

(f)

Let  $r_1^{-1} = x_1$  and  $r_2^{-1} = x_2$ . Rearranging the equation for *b* gives

$$x_2 = \frac{b - x_1}{A}$$

Substituting this into the equation for a gives

$$\left(1+\frac{1}{A}\right)x_1^2-\frac{2b}{A}x_1+\left(\frac{b^2}{A}-\frac{2a}{\Gamma}\right)=0.$$

The first root gives  $x_1 = 0.00526 \text{ nm}^{-1}$  and hence  $x_2 = \frac{b-x_1}{A} = 0.249 \text{ nm}^{-1}$ , which corresponds to  $r_1 = 190 \text{ nm}$  and  $r_2 = 4.01 \text{ nm}$ .

The second root gives  $x_1 = 0.182 \text{ nm}^{-1}$  and hence  $x_2 = \frac{b-x_1}{A} = -0.0621 \text{ nm}^{-1}$ , which corresponds to an unphysical  $r_2 < 0$  and is therefore discarded.

Hence the final answer is  $r_1 = 190 \text{ nm}$  and  $r_2 = 4.01 \text{ nm}$ .

Alternative solution: can equivalently rearrange for

 $x_1 = b - Ax_2$ 

which leads to the quadratic

$$(A^{2} + A)x_{2}^{2} - 2Abx_{2} + \left(b^{2} - \frac{2a}{\Gamma}\right) = 0.$$

This gives rise to the same values of  $x_1$  and  $x_2$  as above.

Correct  $r_1$  and  $r_2$  scores all three marks. One mark for deriving the quadratic equation for  $x_1$  (or equivalently  $x_2$ ), one mark for each correctly calculated radius (full marks for  $r_1$  in the range 180-200 nm and for  $r_2$  in the range 3.95-4.05 nm.

Error carried forward can be given. One mark to be awarded for each radius consistent with the equation for  $x_{1,2}$  given above, evaluated with the values of A, a, and b the student has used. The suggested values of A = 0.5, a = 0.0025 and b = 0.125 give  $r_1$  = 249 nm and  $r_2$  = 4.13 nm.

Total out of 12

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