1 Problem 2: liquified gas

1.1

Simply solving for p=1 bar the equation:

$$log(p/bar) = 3.99 - \frac{443}{(T/K - 0.49)} \tag{1}$$

the boiling temperature is

$$T = 110,43K$$

1.2

The ratio between the energy densities of liquid and gaseous methan is equivalent to the ratio between their densities at a give T and P.
Given the following datas: $V = 4 \times 10^4 m^3$, $T = 298K$, $m = 1.68 \times 10^7\ kg, p = 300bar$, $M_r = 0,016kgmol^{-1}$

$$\frac{\rho_l}{\rho_g} = \frac{m}{V}\frac{RT}{pM_r} = 2,16 \tag{2}$$

1.3

the temperature of trasportation is T= 114 K First of all we have to find out the equilibrium pression of the gas-liquid methane inside the tank, that's to say we must find out the vapour tension above the surface of the liquid. We using equation **??** we obtain $log(pbar^{-1}) = 0,087$. Now looking at the graph we ndividuate the value ΔU making the difference between the U value of pure gas and the U value of pure liquid equilibrium mixture. $\Delta U = 7,3\ kJ$

Indicating with H the enthalpy of vaporization of methane since

$$dH = dU + d(pV) \tag{3}$$

and p is costant, integrating we obtain

$$\Delta H = \Delta U + p(V_g - V_l) \approx \Delta U + pV_g = \Delta U + RT = 8,24 \times 10^3 kJ$$

1.4

The fraction of evaporated methane is

$$x = \frac{n_g}{n_0} = \frac{Pt}{\Delta H n_0} = 7,48 \times 10^{-3}$$

Where $P = 50kW$

1.5

From $T_i = 114K$ we find $log(\bar{p}) = 8,7 \times 10^-2$ From which we derive the initial molar energy of the liquid methane $U_i l = \frac{8,7}{2} \times 10^-2$ given $f'(0) \approx \frac{1}{2}$
Form $p_f = 16,4\ bar$, result $U_f g = 7,8\ kJmol^{-1}$ nad $U_f l = 2,3\ kJmol^{-1}$
Since $dq = dE$ then

$$Pt = \frac{n_o}{3}\Delta U = \frac{n_0}{3}(xU_f g + (1-x)U_f l - U_i l)$$

It follows $x = 0.118$

1.6

The last question is asking us the critical coordinates of methane namely the coordinates where the liquid state still is stable before desappearing to form a supercritical mixture. At the critical point as it is easy to immagine the

$$\Delta H = 0$$

namely no energy is required to pass to the other phase.
Remembering the approximated Clausius Clapeyron relation for gas-liquid transition

$$\frac{dp}{dT} = \frac{\Delta H}{RT^2} \tag{4}$$

the requisite for such a state is

$$\left(\frac{dp}{dT}\right) = 0$$

since the internal energy U is directly proportional to the temperature of a substance $U \alpha T$, $dU = kdT$ applying the chain rule we find

$$\left(\frac{dp}{dU}\right) = 0$$

So we pick up the value where the function of equilibrium in the graph has an orizontal tangent.
Therefore $log(p) = 1,65$ and using eq. **??** $T_c = 190,8K$ and $p_c = 44,66bar$

Solution proposed by Lorenzo Terenzi